Albert Newen and Bernhard Schröder

10 The influence of logic on semantics

Abstract: The aim of this contribution is to investigate the influence of logical tools on the development of semantic theories and vice versa. Pre-19th-century logic was limited to a few sentence forms and their logical interrelations. Modern predicate logic and later type logic, both inspired by investigating the meaning of mathematical sentences, widened the view for new sentence forms and thereby made logic relevant for a wider range of expressions in natural language. In a parallel course of developments the problem of different levels of meaning like sense and reference, or intension and extension were studied and initiated a shift to modal contexts in natural language. Montague bundled in his intensional type-theoretical framework a great part of these development in a unified formal framework which had strong impact on the formal approaches in natural language semantics. While the logical developments mentioned so far could be seen as direct answers to natural language phenomena, the first approaches to dynamic logic did not get their motivation from natural language, but from the semantics of computer programming. Here, a logical toolset was adapted to specific problems of natural language semantics.

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1 Overview

The aim of this contribution is to investigate the influence of logical tools on the development of semantic theories and vice versa. The article starts with an example of Pre-Fregean logic, i.e. Aristotelian syllogistic. This traditional frame of logic has severe limits for a theory of meaning (e.g. no possibility for multiple quantification, no variety of scope). These limitations have been overcome by Frege's predicate logic which is the root of standard modern logic and the basis for the first developments in a formal philosophy of language: We will present Frege's analysis of mathematical sentences and his transfer to the analysis of natural language sentences by introducing a theory of sense and reference. The insufficient treatment of singular terms in Frege's logic is one reason for Russell's theory of definite descriptions. Furthermore, Russell tries to organize semantics without a difference between sense and reference. Carnap introduces the first logical framework for a semantics of possible worlds and shows how one can keep the Fregean semantic intuitions without using the problematic tool of "senses". Carnap develops a systematic theory of intension and extension defining the intension of an expression as a function from possible worlds to the relevant extension. An important formal step was then the invention of modal logic by Kripke. This is the framework for the theory of direct reference of proper names and for the so-called two-dimensional semantics which is relevant to receive an adequate treatment of names, definite descriptions, and especially indexicals. Tarski's formal theory of truth is used by Davidson to argue that truth-conditions are the adequate tool to characterize the meaning of assertions. Although the idea of a truth-conditional semantics is already in the background since Frege, with Davidson's work it became the leading idea for modern semantics.

The second part of the article (starting with section 8) will concentrate on important progresses made in this overall framework of truth-conditional semantics. Montague presented a compositional formal semantics including quantifiers, intensional contexts and the phenomenon of deixis. His ideal was to offer an absolute truth-condition for any sentence. In the next step new formal tools were invented to account not only for the extralinguistic environment but also for the discourse as a core feature of the meaning of utterances. Context-dependency in this sense is considered in approaches of dynamic semantics. Formal tools are nowadays not only used to answer the leading question "What is the meaning of a natural language expression?" In recent developments new logic formalisms are used to answer questions like "How is a convention established?" and "How can we account for the pragmatics of the utterance?" It has become clear that truth-conditional semantics has to be completed by aspects of the environment, social conventions and speaker's intentions to receive an adequate account of meaning. Therefore the latest trend is to offer new formal tools that can account for these features.

2 Pre-Fregean logic

The first system of logic was grounded by Aristotle (see 1992). He organized inferences according to a syllogistic schema which consists of two premises and a conclusion. Each sentence of such a syllogistic schema contains two predicates (F, G), a quantifier in front of the first predicate (some, every) and a negation (not) could be added in front of the second predicate. Each syllogistic sentence has the following structure "Some/every F is/is not G". Then we receive four possible types of sentences: A sentence is universal if it starts with "every" and particular if it starts with "some". A sentence is affirmative if it does not contain a negation in front of the second predicate otherwise it is negative. We receive Tab. 10.1.

Tab. 10.1: Syllogistic types of sentences

NAME	FORM	TITLE
a	Every F is G	Universal Affirmative
i	Some F is G	Particular Affirmative
e	Every F is not G	Universal Negative
o	Some F is not G	Particular Negative

The name of the affirmative syllogistic sentences is due to the Latin word "affirmo". The first vowel represents the universal sentence while the second vowel represents the particular. The name of the negative syllogistic sentences is due to the Latin word "nego" again with the same convention concerning the use of the first and second vowel. As we will see the sequence of vowels is also used to represent the syllogistic inferences. If we introduce the further quantifier "no" we can find equivalent representations but no new propositions. The sentence "No F is G" is equivalent to (e) "Every F is not G" and "No F is not G" is equivalent to (a) "Every F is G". The proposition (e) is intuitively more easily to grasp in the form "No F is G" while proposition (a) is better understandable in the original format "Every F is G". So we continue with these formulations. On the basis of these sentences we can systematically arrange the typical syllogistic inferences, e.g. the inference called "barbara" because it contains three sentences of the form (a).

Tab. 10.2: Barbara

Premise 1 (a):	Every G is H.	abbreviation: GaH
Premise 2 (a):	Every F is G.	abbreviation: FaG
Conclusion (a):	Every F is H.	abbreviation: FaH

Now we can start with systematic variations of the four types of sentences (a, i, e, o). The aim of Aristotle was to select all and only those inferences which are valid. Given the same structure of the predicates in the premises and the conclusion only varying the kind of sentence we receive e.g. the valid inferences of Tab. 10.3.

Tab. 10.3: Same predicate structure, varying types of sentences

Barbara	Darii	Ferio	Celarent
Every M is H	Every M is H	No M is H	No M is H
Every F is M	Some F are M	Some F is M	Every F is M
Every F is H	Some F are H	Some F is not H	No F is H

To present the complete list of possible syllogistic inferences we have to account for different kinds of predicate positions in the inference. We can distinguish four general schemata including the one we already had presented so far. Our first schema has the general structure (I) and we also receive the other structures (II to IV) in Tab. 10.4.

Tab. 10.4: Predicate structures

I.	M	H	II.	H	M	III.	M	H	IV.	H	M
	F	M		F	M		M	F		M	F
	F	H		F	H		F	H		F	H

For each general format we can vary the kinds of sentences that are involved in the way presented above. This leads to all possible syllogistic inferences in the Aristotelian logic. While making this claim we are ignoring the fact that Aristotle already worked out a modal logic, cf. Nortmann (1996). Concentrating on non-modal logic we have presented the core of the Aristotelian system. Although it was an ingenious discovery in ancient times, the Aristotelian system has its strong limitations: Strictly speaking, there is no space in syllogistic inferences for (a) singular terms, (b) existence claims (like "Trees exist") and there are only very limited possibilities for quantification (see section on Frege's progress). Especially, there is no possibility for multiple uses of quantifiers in one sentence. This is the most important progress which is due to the predicate logic essentially developed by Gottlob Frege. Before we present this radical step into modern logic, we shortly describe some core ideas of G. W. Leibniz, who invented already some influential ideas on the way to modern logic.

Leibniz is well-known for introducing the idea of a *calculus of logical inferences*. He introduced the idea that the syntax of the sentences mirrors the logical structure of the thoughts expressed and that there can be defined a purely syntactic procedure of proving a sentence. This leads to the modern understanding of a syntactic notion of proof which ideally allows for all sentences to decide simply on the basis of syntactic transformations whether they are provable or not. The logic systems developed by Leibniz are essentially advanced compared to the Aristotelian syllogistic. It has been shown that his logic is equivalent to the Boolean logic, i.e. the monadic predicate logic, see Lenzen (1990). Furthermore, Leibniz introduced a calculus of concepts defining concept identity, inclusion, containment and addition, see Zalta (2000) and Lenzen (2000). He reserved a special place for individual concepts. Since his work had almost no influence on the general development of logic the main ideas are only mentioned here. Ignoring a lot of interesting developments (e.g. modal systems) we can characterize a great deal of the logical systems initiated by Aristotle until the 19th century by the square of opposition (cf. article 8 [this volume] (Meier-Oeser) *Meaning in pre 19th-century thought*).

The square of opposition (see Fig. 10.1) already involves the essential distinction between different understandings of "oppositions": A contradiction of a sentence is an external negation of sentence "It is not the case that ..." while the contrary involves an "internal" negation. What is meant by an "internal" negation can only be illustrated by transforming the syllogistic sentences into modern predicate logic. The most important general features in the traditional understanding are the following: (i) From two contradictory sentences one must be false

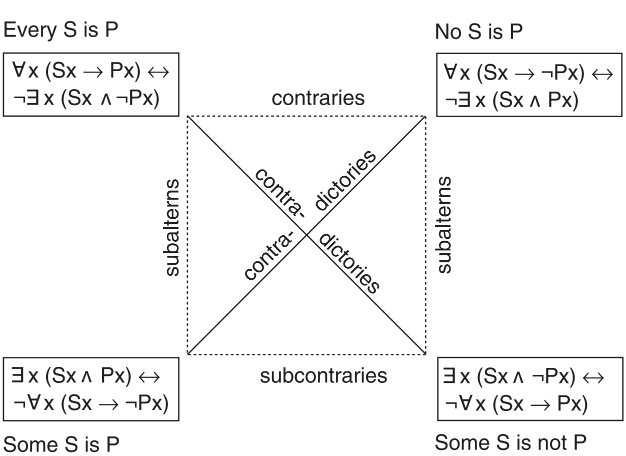

Fig. 10.1: Square of oppositions

and one be true. (ii) For two contrary sentences holds that they cannot both be true (but they can both be false). (iii) For two subcontrary sentences holds that they cannot both be false (but they can both be true).

A central problem pointed out by Abelard in the Dialectica (1956) is *presupposition of existential import:* According to one understanding of the traditional square of opposition it presupposes that sentences like “Every F is G” or “Some F is G” imply “There is at least one thing which is F”. This is the so-called existential import condition which leads into trouble. The modern transformation of the syllogistic sentences into predicate logic, as included into the figure above, does not involve the existential presupposition. Let us use an example: On the one hand, “Some man is black” implies that at least one thing is a man, namely the man who has to be black if “Some man is black” is true. On the other hand, “Some man is not black” also implies that something is a man, namely the man who is not black if “Some man is not black” is true. But these are two subcontrary sentences, i.e. according to the traditional view they cannot both be false; one has to be true. Therefore (since both imply that there is a thing which is a man) it follows that men exist. In such a logic the use of a predicate F in a sentence “Some F...” presupposes that F is non-empty (simply given the meaning of F and the traditional square of opposition), i.e. there are no empty predicates. But of course (as Abelard points out) surely men might not exist. This observation leads to the modern square of opposition which uses the reading of sentences in predicate logic and leaves out the relations of contraries and subcontraries and only keeps the relation of the contradictories. The leading intuition to avoid the problematic consequence mentioned above is the claim that meaningful universal sentences like “Every man is mortal” do not imply that men exist. The existential import is denied for both predicates in universal sentences. Relying on the interpretation of particular sentences like “Some men are mortal” according to modern predicate logic, the truth of the particular sentences just means that there is at least one man which is mortal. If we want to allow nonempty predicates then we receive the modern square of opposition.

3 Gottlob Frege’s progress

Frege was a main figure from two points of view: He introduced a modern predicate logic and he also developed the first systematic modern philosophy of language by transferring his insights in logic and mathematics into a philosophy of language. His logic was first developed in the “Begriffsschrift” (1879, in the following shortly:

BS), see Frege (1977). He used a special notation to characterize the content of a sentence by a horizontal line (the content line) and the act of judging the content by a vertical line (the judgement line).

$$\vdash A$$

This distinction is a clear precursor of the distinction between illocution (the type of speech act) and proposition (the content of the speech act) in Searle's speech act theory.

Frege's main aim was to clarify the status of arithmetic sentences. Dealing with a mathematical expression like „3^2“ Frege analyzes it into the functional expression „$(\ )^2$“ and the argument expression „3“. The functional expression refers to the function of squaring something while the argument expression refers to the number 3. An essential feature of the functional expression is its being unsaturated, i.e. it has an empty space that needs to be filled by an argument expression to constitute a complete sentence. The argument expression is saturated, i.e. it has no space for any addition. Frege transferred this observation into the philosophy of language: Predicates are typical expressions which are unsaturated, while proper names and definite descriptions are typical examples for saturated expressions. Predicates refer to concepts while proper names and definite descriptions refer to objects. Since predicates are unsaturated expressions which need to be completed by a saturated expression, Frege defines concepts (as the reference of predicates) as functions which – completed by objects (as the referents of proper names and other singular terms) – always have a truth value (Truth or Falsity) as result. The truth-value is the reference of the sentence composed of the proper name and the predicate. By analogy from mathematical sentences Frege starts to analyze sentences of natural language and develops a systematic theory of meaning. Before outlining some basic aspects of this project, we first introduce the idea of a modern system of logic. Frege developed the following propositional calculus (for a detailed reconstruction of Frege's logic see von Kutschera 1989, chap. 3):

AXIOMS:

(1) a. $A \rightarrow (B \rightarrow A)$
b. $(C \rightarrow (B \rightarrow A)) \rightarrow ((C \rightarrow B) \rightarrow (C \rightarrow A))$
c. $(D \rightarrow (B \rightarrow A)) \rightarrow (B \rightarrow (D \rightarrow A))$
d. $(B \rightarrow A) \rightarrow (\neg A \rightarrow \neg B)$
e. $\neg\neg A \rightarrow \mathrm{A}$
f. $\mathrm{A} \rightarrow \neg\neg\mathrm{A}$

A RULE OF INFERENCE, which allows to derive theorems by starting with two axioms or theorems:

(2) $A \rightarrow B, A \vdash B$

If we add an axiom and a rule we receive a system of predicate logic that is complete and consistent. Frege suggested the following axiom:

(3) $\forall x A[x] \rightarrow A[a]$ (BS: 51).

The additionally relevant rule of inference was not explicitly marked by Frege but presupposed implicitly:

(4) $A \rightarrow B[a] \vdash A \rightarrow \forall x B[x]$, if „$a$" is not involved in the conclusion (BS: 21).

Frege tried to show the semantic consistency of the predicate calculus (which was then intensily debated) but he did not try to prove the completeness since he lacked the notion of interpretation to develop such a proof (von Kutschera 1989: 34). A formal system of axioms and rules is complete for first-order predicate logic (FOPL) if all sentences logically valid in FOPL are derivable in the formal system. The completeness proof was for the first time worked out by Kurt Gödel (1930). Frege already included second-order predicates into his system of logic. The interesting fact that second-order predicate logic is incomplete was for the first time shown by Kurt Gödel (1931).

One of the central advantages of modern predicate logic for the development of semantics is the fact that we can now use as much quantifiers in sequence as we want. We have of course to take care of the meaning of quantifiers given the sequence. The following sentences which cannot be expressed in the system of Aristotelian syllogism can be nicely expressed by the modern predicate logic using "L" as a shorthand for the two-place predicate "( ) loves ( )".

(5) Everyone loves everyone: $\forall x \forall y L(x, y)$

Using mixed quantifiers their sequence becomes relevant:

(6) a. Someone loves everyone: $\exists x \forall y L(x, y)$

b. Everyone loves someone: $\forall x \exists y L(x, y)$
[This can be a different person for everyone]

c. Someone is loved by everyone: $\exists y \forall x L(x, y)$
[There is (at least) one specific human being who is loved by all human beings]

d. Everyone is loved by someone: $\forall y \exists x L(x, y)$

e. Someone loves someone: $\exists x \exists y L(x, y)$
[There is (at least) one human being who loves (at least) one human being]

Frege's philosophy of language is based on a principle of compositionality (cf. article 6 [this volume] (Pagin & Westerståhl) *Compositionality*), i.e. the principle that the value of a complex expression is determined by the values of the parts plus its composition. He developed a systematic theory of sense and reference (cf. article 3 [this volume] (Textor) *Sense and reference*). The reference of a proper name is the designated object and the reference of a predicate is a concept while both determine the reference of the sentence, i.e. the truth-value. Given this framework of reference it follows that the sentences

(7) The morning star is identical with the morning star.

and

(8) The morning star is identical with the evening star.

have the same reference, i.e. the same truth-value: The truth-value is determined by the reference of the name and the predicate. Each token of the predicate refers to the same concept and the two names refer to the same object, the planet Venus. But sentence (7) is uninformative while sentence (8) is informative. Therefore, we need a new aspect of meaning to account for the informativity: the sense of an expression. The sense of a proper name is a mode of presentation of the designated object, i.e. "the evening star" expresses the mode of presentation characterized as the brightest star in the evening sky. Furthermore the sense of a sentence is a thought. The latter (in the case of simple sentences like "Socrates is a philosopher") is constituted by the sense of a predicate "( ) is a philosopher" and the sense of the proper name "Socrates". Frege defines the sense of an expression in general as the mode of presentation of the reference. To develop a consistent theory of sense and reference Frege introduced different senses for one and the same expression in different linguistic contexts, e.g. indirect speech (propositional attitude ascriptions) or quotations are contexts in which the sense of an expression changes. Frege's philosophy of language has at least two major problems: (1) the necessity of an infinite hierarchy of senses to account for the recursive syntactic structure (John believes that Mary believes that Karl believes .....) and (2) the problem of indexical expressions (it is accounted for in two-dimensional semantics and dynamic semantics, see below).

4 Bertrand Russell's criticism and his theory of definite descriptions

Russell (1903, partly in cooperation with Whitehead (Russell & Whitehead 1910–1913)) also developed himself both a system of logic and a philosophy of language in contrast to Frege such that we nowadays speak of Neo-Fregean and Neo-Russellian theories of meaning. Let us first have a look at Russell's logical considerations. Russell developed his famous paradox which was a serious problem for Frege because Frege presupposes in his system that he could produce sets of sets in an unconstrained manner. But if there are no constraints we run into Russell's paradox: Let R be the set of all sets which are not members of themselves. Then R is neither a member of itself nor not a member of itself. Symbolically, let $R := \{x : x \notin x\}$. Then $R \in R$ iff $R \notin R$. To illustrate the consideration: If R is a member of itself it must fulfill the definition of its members, i.e. it must not be a member of itself. If R is not a member of itself then it should not fulfill the definition of its members, i.e. it must be a member of itself. When Russell wrote his discovery in a letter to Frege who was just completing *Grundlagen der Arithmetik* Frege was despaired because the foundations of his system were undermined. Russell himself developed a solution by introducing a *theory of types* (1908). The leading idea is that we always have to clarify those objects to which the function will apply before a function can be defined exactly. This leads to a strict distinction between object language and meta-language: We can avoid the paradox by avoiding self-references and this can be done by arranging all sentences (or, equivalently, all propositional functions) into a hierarchy. The lowest level of this hierarchy will consist of sentences about individuals. The next lowest level will consist of sentences about sets of individuals. The next lowest level will consist of sentences about sets of sets of individuals, and so on. It is then possible to refer to all objects for which a given condition (or predicate) holds only if they are all at the same level, i.e. of the same type. The theory of types is a central element in modern theory of truth and thereby also for semantic theories. Russell's contribution to the philosophy of language is essentially connected with his analysis of definite descriptions (Russell 1905). The meaning of the sentence "The present King of France is bald" is analyzed as follows:

1. there is an x such that x is the present King of France ($\exists x(Fx)$)
2. for every x that is the present King of France and every y that is the present King of France, x equals y (i.e. there is at most one present King of France) ($\forall x(Fx \rightarrow \forall y\,(Fy \rightarrow y = x))$)
3. for every x that is the present King of France, x is bald. ($\forall x(Fx \rightarrow Bx)$)

Since France is no longer a kingdom, assertion 1. is plainly false; and since our statement is the conjunction of all three assertions, our statement is false.

Russell's analysis of definite descriptions involves a strategy to develop a purely extensional semantics, i.e. a semantic theory that can characterize the meaning of sentences without introducing the distinction between sense and reference or any related distinction of intensional and extensional meanings. Definite descriptions are analyzed such that there remains no singular term in the reformulation and ordinary proper names are according to Russell's theory hidden definite descriptions. His strategy eliminates singular terms with only one exception: He needs the basic singular term "this/that" to account for our speech about sense-data (Russell 1910). Since he takes an acquaintance relation with sense-data (and also with universals) including a sense-data ontology as a basic presupposition of his specific semantic approach, the only core idea that survived in modern semantics is his logical analysis of definite descriptions.

5 Rudolf Carnap's theory of extension and intension: Relying on possible worlds

Since Russell's project was idiosyncratically connected with a sense-data theory it was for the great majority of scientists not acceptable as a purely extensional project. The extensional semantics had to wait until Davidson used Tarski's theory of truth as a framework to characterize a new extensional semantics. Meanwhile it was Rudolf Carnap who introduced the logic of extensional and intensional meanings to modernize Frege's twofold distinction of semantics. The central progress was made by introducing the idea of possible worlds into logics and semantics: The actual world is constituted by a combination of states of affaires which are constituted by objects (properties, relations etc.). If at least one state of affairs is changed we speak of a new possible world. If the world consists of basic elements which constitute states of affaires then the possible combinations of these elements allow us to characterize all possible states of affaires. Thereby we can characterize all possible worlds since a possible world can be characterized by a class of states of affaires that is realized in this world. Using this new instrument of possible worlds Carnap introduces a systematic theory of intension. His notion of intension should substitute Frege's notion of sense and thereby account for the informational content of a sentence. His notion of extension is closely connected to Frege's notion of reference: The extension of a singular term is the object referred to by the use of the term, the extension of a predicate is the property referred to and the extension of a complete assertive sentence is its truth-value. The intension which has to account for the informational content is characterized as a function from possible worlds to the relevant extensions. In the case of a singular term the intension is a function from possible worlds (p.w.) to the object referred to in the relevant possible world. In the same line you receive the intension of predicates (as function from p.w. to sets or n-tuples) and of sentences (as functions from p.w. to truth-values) as shown in Tab. 10.5.

Tab. 10.5: Carnap's semantics of possible worlds

	Extension	**Intension**
singular terms	objects	individual concepts
predicates	sets of objects and n-tuples of objects	properties
sentences	truth-values	propositions

A principle limitation of a semantic of possible worlds is that you cannot account for so-called hyperintensional phenomena, i.e. one cannot distinguish the meaning of two sentences which are necessarily true (e.g. two different mathematical claims) because they are simply characterized by the same intension (matching each p.w. onto the value "true").

6 Willard V. O. Quine: Logic, existence and propositional attitudes

How should we relate quantifiers with our ontology? Quine (1953) is famous for his slogan "To be is to be the value of a bound variable". Quantifiers "there is (at least) an x ($\exists x$)", "for all x ($\forall x$)" are the heart of modern predicate logic which was already introduced by Frege (s. above). For Quine the structure of the language determines the structure of the world: If the language which is necessary to receive the best available complete description of the world contains several existential and universal quantifications then these quantifications at the same time determine the objects, properties etc. we have to presuppose. Logic, language and ontology are essentially connected according to this view. Although Quine's special views about connecting logic, language and world are very controversial nowadays the core of the idea of combining quantificational and ontological claims is widely accepted.

Another problem that is essentially inspired by the development of logic is the analysis of propositional attitude ascriptions. Quine established the following standard story:

> There is a certain man in a brown hat whom Ralph has glimpsed several times under questionable circumstances on which we need not enter here; suffice it to say that Ralph suspects he is a spy. Also there is a gray-haired man, vaguely known to Ralph as rather a pillar of the community, whom Ralph is not aware of having seen except once at the beach. Now Ralph does not know it, but the men are one and the same. Can we say of this man (Bernard J. Ortcutt, to give him a name) that Ralph believes him to be a spy? If so, we find ourselves accepting a conjunction of the type:
>
> (9) w sincerely denies '.....' . w believes that .....
>
> as true, with one and the same sentence in both blanks. For, Ralph is ready enough to say, in all sincerity, 'Bernard J. Ortcutt is no spy.' If, on the other hand, with a view to disallowing situations of the type (9), we claim simultaneously that
>
> (10) Ralph believes that the man in the brown hat is a spy.
>
> (11) Ralph does not believe that the man seen at the beach is a spy.
>
> then we cease to affirm any relationship between Ralph and any man at all.[...] 'believes that' becomes, in a word, referentially opaque.
>
> (Quine 1956: 179, examples renumbered)

In line with Russell, Quine starts to analyze the cognitive situation of Ralph by distinguishing two readings of the sentence

(12) Ralph believes that someone is a spy.

namely:

(13) Ralph believes [$\exists x$(x is a spy)]

(14) $\exists x$(Ralph believes [x is a spy])

Quine calls (13) the notional and (14) the relational reading of the original sentence which is at the first glance parallel to the traditional distinction between *de dicto* (13) and *de re* (14) reading. But he shows that the difference of these two readings is not sufficient to account for Ralph's epistemic situation as characterized with the sentences (10) and (11). Intuitively Ralph has a *de re* reading in both cases, one of the man on the beach and the other of a person wearing a brown hat. The transformation into *de re* readings leads to:

(15) $\exists x$(Ralph believes [x is a spy]) (out of (10))

(16) $\exists x$(Ralph does not believe [x is a spy]) (out of (11))

Since both extensional quantifications are about the same object, we receive the combined sentence which explicitly attributes contradictory beliefs to Ralph:

(17) $\exists x$(Ralph believes [x is a spy] $\wedge$ Ralph does not believe [x is a spy])

To avoid this unacceptable consequence Quine suggests that the ambiguity of the belief sentences cannot be accounted for by a distinction of the scopus of the quantifier (leading to *de re* and *de dicto* readings) but by a systematic ambiguity of the belief predicate: He suggests to distinguish a two-place predicate "Believe^2 (subject, proposition)" and a three-place predicate "believe^3 (subject, object-of-belief, property)".

(18) Believe^2 (Ralph, that the man with the brown hat is a spy)

(19) believe^3 (Ralph, the man with the brown hat, spy-being)

This distinction is the basis for Quine's further famous claim: We are not allowed to quantify into propositional attitudes (i.e. implying (14) from (18)): if we have interpreted a sentence such that the 'believe' predicate is used intensionally (as a two-place predicate) then we cannot ignore that and we are not allowed to change the reading to the sentence into one using a three-place predicate. We are not allowed to change from a notional reading (18) into a relational reading (19) and vice versa. This line of strategy was further improved e.g. by Kaplan (1969) and Loar (1972). It definitely made clear that we cannot always understand the belief expressed by a belief sentence simply as a relation between a subject and a proposition. Sometimes it has to be understood differently. Quine's consequence is a systematic ambiguity of the predicate "believe". This is problematic since it leads also to four-place, five-place predicates etc. (Haas-Spohn 1989: 66): for each singular term which is used in the scopus of the belief ascription we have to distinguish a notional and a relational reading. Therefore Cresswell & von Stechow (1982) suggested an alternative view which only needs to presuppose a two-place predicate "believe" but therefore changes the representation of a proposition: A proposition is not completely characterized by a set of possible worlds (according to which the relevant state of affaires is true) but in addition by a structure of the components of the proposition. Structured propositions are the alternative to a simple possible world semantics to account for propositional attitude ascriptions.

7 Necessity and direct reference: The two-dimensional semantics

The development of modal logic essentially put forward by Saul A. Kripke had strongly influenced the semantical theories. The basic intuition the modal logic started with is rather straightforward: Each entity is necessarily identical with itself (and necessarily different from anything else). Kripke (1972) shows that there are sentences which express a necessary truth but nevertheless are *a posteriori*: “Mark Twain is identical with Samuel Clemens”. Since “Samuel Clemens” is the civil name of Mark Twain the sentence expresses a self-identity but it is not known *a priori* since knowing that both names refer to the same object is not part of standard linguistic knowledge. There are also sentences which express contingent facts but which can be known to be true a priori, e.g. “I am speaking now”. If I utter the sentence it is *a priori* graspable that it is true but it is not a necessary truth since otherwise I would be a necessary speaker at this timepoint (but of course I could have been silent). To account for the new distinction between epistemic dimension of *a priori/a posteriori* and the metaphysical dimension of necessary/contingent Kripke introduced the theory of direct reference of proper names and Kaplan (1979/1989) introduced the two-dimensional semantics. Since Kaplan’s theory of characters is nowadays a standard framework to account for names and indexicals we shortly introduce the core idea: We have to distinguish the utterance context which determines a proposition which is expressed by uttering a sentence and the circumstance of evaluation which is the relevant possible world according to which this proposition will be evaluated as true or false. We can illustrate this two-step approach as in Fig. 10.2:

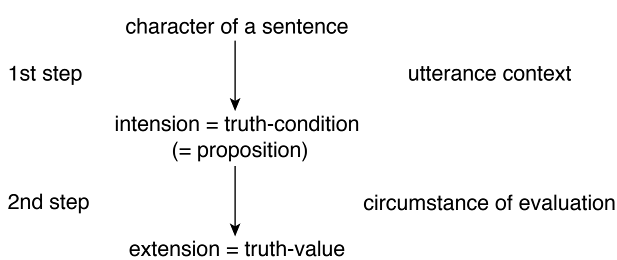

Fig. 10.2: Two-dimensional semantics

A character of a sentence is a function from possible utterance contexts to truth-conditions (propositions) and these truth-conditions are then evaluated relative to the circumstances of evaluation. Especially in the cases of indexicals we can demonstrate the two-dimensional semantics: Let us investigate the sentence

"I am a philosopher" according to three different utterance contexts w_0, w_1 und w_2 while in each world there is a different speaker: in w_0 Cicero is the speaker of the utterance, in w_1 Caesar and in w_2 Augustus. Furthermore it is in w_0 the case that Cicero is a philosopher while Caesar and Augustus are not. In w_1 Caesar and Augustus are philosophers while Cicero isn't. In w_2 no one is a philosopher (poor world!). The three worlds function both as utterance contexts and as circumstances of evaluation but of course different facts are relevant in the two different functions. Now the utterance contexts are represented veritically while the circumstances of evaluation are represented horizontally. Then the sentence "I am a philosopher" receives the character shown in Tab. 10.6.

Tab. 10.6: Character of "I am a philosopher"

Utterance Contexts	**Circumstances of evaluation**			**Truth conditions**
	w_0	w_1	w_2	
w_0	*w*	*f*	*f*	⟨Cicero; being a philosopher⟩
w_1	*f*	*w*	*f*	⟨Caesar; being a philosopher⟩
w_2	*f*	*w*	*f*	⟨Augustus; being a philosopher⟩

Each line represents the proposition that is determined by the sentence relative to the utterance contexts and this proposition receives a truth-value for each circumstance of evaluation. The character of the sentences has in principle to be represented for all possible worlds not only for the three ones selected above. This instrument of a "character" of a sentence is useable for all expressions of a natural language. Such it is a principle improvement and enlargement in the formal semantics that contains Carnap's theory of intensions as a special case.

8 Montague-Semantics: Compositionality revisited

Frege claimed in his principle of compositonality that the meaning of a complex expression is a function of the meanings of the expression parts and their way of composition (see section 3.). He regarded every complex expression as composed of a saturated part and a non-saturated part. The semantic counterparts are objects and functions. Frege transfers the syntactic notion of an expression which needs some complement to be a complete and well-formed expression to the semantic realm: Functions are regarded as incomplete and non-saturated. This leads him to the ontological view that functions are not "objects" ("Gegenstände").

This is Frege's term for any entitity that can be a value of a meaning function mapping expressions to their extensions. Functions, however, can be reified as "Werthverläufe" (courses-of-values), e.g. by combining their expressions with the expression *the function*, but in Frege's ontology, the "Werthverläufe" are distinct from the functions themselves.

Successors of Frege did not follow him in this ontological respect of his semantics. In Tarskian semantics one-place predicates are usually mapped to sets of individuals, two-place predicates to sets of pairs of individuals etc. N-place functions are regarded as special kinds of (n-1)-place relations. This approach allowed to make explicit the meaning of Frege's non-saturated expressions and to give a precise compositional account of the meanings of expressions of predicate logic in form of a recursive definition. But for every type of composition, like connecting formulae, applying a predicate to its arguments, or prefixing a quantifier to a formula, distinct forms of meaning composition were needed.

The notion of compositionality could be radically simplified by two developments: The application of type theory and lambda abstraction. Type theory goes back to Russell and was developed to avoid paradoxical notions as sets not containing themselves, see above sec. 4. There are a lot of versions of type theory, but in natural language semantics usually a variant is used which starts with a number of basic types, e.g. the type of objects (entities) e and the type of truth values t for an extensional language, and provides a type of functions $\langle T_1, T_2 \rangle$ for every type T_1 and every type T_2, i.e. the type of functions from entities of type T_1 to entities of type T_2. Sometimes the set of types is extended to types composed of more than two types. But any such system can easily be reduced to this binary system. Predicates can now be viewed as expressions of type $\langle e, t \rangle$, i.e. as functions from objects to truth values, because they yield a true sentence if an argument is filled in that refers to an instance of the predicate, otherwise they yield a false sentence. That just means that we take the characteristic function of the predicate extension as its meaning. A characteristic function of a set maps its members to the truth value true, its non-members to false.

In the same sense the negation operator is of type $\langle t, t \rangle$, i.e. a function from a truth-value to a truth-value (namely to the opposite one); binary sentence connectives are of type $\langle t, \langle t, t \rangle\rangle$, i.e. a function taking the first argument and yielding a function which takes the second argument and then results in a truth value.

Frege had already recognized that first-order quantifiers (as the existential quantifier *something*) are just second-order predicates, i.e. predicates applicable to first-order predicates. The application of an existential quantifier to a one-place first-order predicate is true if the predicate is non-empty. Therefore the existential quantifier can be regarded as a second-order predicate which has non-empty first-order predicates as instances, it is of type $\langle\langle e, t \rangle, t \rangle$. The semantics of the universal quantifier is analoguous: It yields the truth value true for a first-order predicate which has the whole universe of discourse as its extension.

A type problem arises with this view for predicates of arity greater than one: Their type does not fit to the quantifier. The predicate *love*, e.g. is of type $\langle e, \langle e, t\rangle\rangle$ because if we add one object as an argument we receive a one-place predicate as introduced above. In the sentence

(20) Everybody loves someone.

one of the two quantifiers has to be composed with *love* in a compositional approach. Let us assume *someone* is this quantifier. Then its meaning of type $\langle\langle e, t\rangle, t\rangle$ has to be applied to the meaning of *love*, leading to a type clash, because the quantifier needs a type $\langle e, t\rangle$ as its argument while the predicate *love* is of a different type. Analoguous problems arise for complex formulae. How can the meaning of the two-place predicate love be transformed into the type required?

The type theory needs an extension by the introduction of a lambda-operator, borrowed from Alonzo Church's lambda calculus which was developed in Church (1936). The lambda operator is used to transform a type t expression into an expression of $\langle T_1, t\rangle$ depending on the variable type T_1. Let e.g. x be a variable of type e and P be of type t, then $\lambda x[P]$ has type $\langle e, t\rangle$, i.e. is a one-place first-order predicate. If we consider *love* as a two-place first-order predicate and love(x, y) as an expression of type t with two free variables, then λx[love(x, y)] is an expression of type $\langle e, t\rangle$. This is the type required by the quantifier. The variable x is bound to the lambda operator and y is free in this expression. If we now take *someone* as a first-order quantifier, which has type $\langle\langle e, t\rangle, t\rangle$, then someone($\lambda x$[love($x$, y)]) is an expression of type t again with free variable y. This can be made a predicate of type $\langle e, t\rangle$ by using the same procedure again: λy[someone(λx[love (x, y)])], which can be used as an argument of a further quantifier *everybody*. We receive the following new analysis of (20):

(21) everybody(λy[someone(λx[love(x, y)])])

The semantics of a lambda expression $\lambda x[P]$ is defined as the characteristic function which yields *true* for all arguments which would make P true, if they were taken as assignments to x, false for the others. With this semantics we get the following logical equivalences which we used implicitly in the above formalizations:

α-CONVERSION

(22) $\lambda x[P] \equiv \lambda y[P[y/x]]$

where $P[y/x]$ is the same as P besides the fact that all occurences of x which are free in P are changed into y. α-conversion is just a formal renaming of variables.

β-REDUCTION

(23) $\lambda x[P](a) \equiv P[a/x]$

where $P[a/x]$ is the same as P besides the fact that all occurences of x which are free in P are changed into a. This, however, may be false in some non-extensional contexts, if a is a non-rigid designator. If we take

(24) Ralph believes that x is a spy.

as P then the *de re* reading of (10) is $\lambda x[P](a)$. a is interpreted outside the non-extensional context of P and its factual denotation is taken as its extension. $P[a/x]$, however, is the *de dicto* reading because a is interpreted within the belief-context. For intensional contexts these differences are treated in the intensional theory of types, see section 10 below.

η-CONVERSION

(25) $\lambda x[P(x)] \equiv P$

where x does not occur as a free variable in P. η-conversion is needed for the conversion between atomic predicates and λ-expressions.

In this type-theoretical view, a lot of other linguistic expression types can easily be integrated. Adjectives, which are applied to nouns of type $\langle e, t\rangle$, can be seen as type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$, e.g. *tasty* is a modifier which takes predicates, expressed by nouns, like *apple* and yields new predicates, like *tasty apple*.

Modifiers in general, like adverbs, prepositional phrases and relative and adverbial clauses, are regarded as type $\langle T_1, T_1\rangle$, because they modify the meaning of their argument, but this results in an expression of the same type. This also mirrors that modifiers can be applied in an iterated manner, as *red tasty apple*, where *red* further modifies the predicate *tasty apple*.

The compositionality of meaning got a very strict interpretation in Montague's work. The type-theoretic semantics was accompanied by a syntactic formalism whose expression categories could be directly mapped onto semantic types, called categorial grammar, which was based on ideas by Kasimierz Ajdukiewicz in the mid-1930s and Yehoshua Bar-Hillel (1953).

Complex semantic types, i.e. semantic types needing some complemetation, like $\langle T_1, T_2\rangle$ for some types T_1 and T_2, have their counterpart in complex syntactic categories like S_2/S_1 and $S_2\backslash S_1$ which need a complementation by S_1 to yield the syntactic category S_2. Given a syntactic category S_2/S_1 a complement of type S_1 has to be added to the right, in case of $S_2\backslash S_1$ to the left. Let e.g. N be the syntactic category of a noun and DP be the category of a determiner phrase, then DP/N is the category of the determiner in the examples above. The interaction of Montague's syntactic and semantic conception results in the requirement that the meaning of a complex expression can be recursively decomposed into function-argument-pairs which are always expressed by syntactically immediately adjacent constituents.

Categorial grammars describe the same class of languages as context free grammars, and they are subject to the same problems when applied to natural languages. Although most phenomena in natural languages can in principle be represented by a context free grammar, and therefore by a categorial grammar, too, both formalisms lead to quite unintuitive descriptions when applied to a realistic fragment of natural language. Especially with regard to semantic compositionality discontinous constituents require a quite unintuitive multiplication of categories.

The type theoretic view of nouns and noun phrases has also consequences for the semantic concept of quantifying expressions. Determiners like *all* or *two* as parts of determiner phrases will have to be assigned a suitable type of meaning.

9 Generalized quantifiers

As mentioned above, already Frege regarded quantifiers as second- or higher-order predicates. But Frege himself did not go the step from this insight to the consideration of other quantifiers than the universal and the existential ones. Without being aware of Frege's concept of quantifiers the generalized view on quantifiers by Mostowski (1957) and Lindström (1966) pathed the way to a generalized theory of quantifiers in linguistics around 1980. This allowed for a proper treatment of syntactically first-order quantifying expressions whose semantics was not expressible in first-order logic. e.g. *most*.

Furthermore, now, noun phrases which had no mere anaphoric function, could be interpreted as a quantifier, consisting of the determiner, e.g. *all*, which specifies the basic quantifier semantics, and the noun, e.g. *women*, or a noun-like expression which restricts the quantification. The determiner is a function of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ taking the restrictor as argument and resulting in a generalized quantifier, e.g. all(woman) for the noun phrase *all women*. This generalized quantifier is (the characteristic function of) a second-order predicate which has all (charateristic functions of) first-order predicates as its instances which are true for all women. As the determiner designates the principle function in such a noun phrase some linguists prefer the term *determiner phrase*.

Generalized quantifiers can be studied with respect to their monotonicity properties. Let *Q* be a generalized quantifier and *Q*(*P*) be true. Then it can be the case – depending on *Q*'s semantics – that *Q*(*P*') is always true if

(A) the extension of *P*' is a subset of the extension of *P* or
(B) the extension of *P* is a subset of the extension of *P*'

In the first case, we call *Q monotone decreasing* or *downward entailing*, in case (B) *monotone increasing* or *upward entailing*. An example of the first quantifier type is *no women*, an example of the second type *(at least) two women*, cf. e.g. the entailment relations between sentences the following. (26a) entails (26b), and (27a) entails (27b).

(26) a. At least two women worked as extremely successful CEOs.
b. At least two women worked as CEOs.

(27) a. No women worked as CEOs.
b. No women worked as extremely successful CEOs.

While quantifiers also expressible in first-order logic, like those in the examples above, always show one of the monotonicity properties, there are other generalized quantifiers which do not. Numerical expressions providing a lower and an upper bound for a quantity, like exact numbers, are examples for non-montonic quantifiers. If we replace *at least two women* with *exactly two women* in the examples above, the entailment relations between the sentences disappear.

Mononicity of quantifiers seems to play an important role in natural language although not all quantifiers are monotonic themselves. But it is claimed that all simple quantifiers, i.e. one-word quantifiers or quantifiers of the form determiner + noun, are expressible as conjunctions of monotonic quantifiers. E.g. *exactly three Ps* is equivalent to the conjunction *at least three Ps and no more than three Ps*, the first conjunct (at least three Ps) being upward monotonic and the latter (no more than three Ps) being downward monotonic. There are, of course, quantifiers not bearing this property, and they are expressible in natural language, cf. *an even number of Ps*, but there does not seem to be any natural language which reserves a simple lexical item for such a purpose. The theory of generalized quantifiers therefore raises empirical questions about language universals.

Similar considerations on entailment conditions can be applied to the first argument of the determiner. For some determiners D it might be the case that $D(R)(P)$ entails $D(R')(P)$ always if

(A′) the extension of R' is a subset of the extension of R or
(B′) the extension of R is a subset of the extension of R'.

In the second case the determiner is called persistent, while in the first it is called antipersistent. Consider the entailment relations between the sentences below. Here, (28a) entails (28b), and (29a) entails (29b).

(28) a. Some extremely successful female CEOs smoke.
b. Some female CEOs smoke.

(29) a. All female CEOs smoke.
b. All extremely successfull female CEOs smoke.

It is easy to see that *some* is persistent while *all* is antipersistent. All combinations of monotonicity and persistence/antipersistence are realized in natural languages. Tab. 10.7 shows some examples. Note that the quantifiers of the square of opposition are part of this scheme.

Tab. 10.7: Monotonicity and (anti-)persistence of quantifiers

	upward monotonic	downward monotonic
antipersistent	all, every	no, at most three
persistent	some, (at least) three	not all

And the relations of being contradictory and (sub-)contrary (see sec. 2) in the square of oppositions are mirrored by negations of the whole determiner-governed sentence or the second argument: $\neg D(R, P)$ is contradictory to $D(R, P)$, while $D(R, \neg P)$ is (sub-)contrary to $D(R, P)$ (we use $\neg P$ as short for $\lambda x[\neg P(x)]$.).

Analyzing quantified expressions as structures consisting of a determiner, a restrictor and a quantifier scope provides us with a relational view of quantifiers. The determiner type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ is that of a second-order two-place relation. Besides the logical properties of the argument positions discussed above, there are a number of interesting properties regarding the relation of the two arguments. One of them is conservativity. A determiner is conservative iff it is always the case that

$$D(R, P) \equiv D(R, P \wedge R)$$

(where we use $P \wedge R$ as the conjunction of the predicates P and R, more precisely $\lambda x\ [P(x) \wedge R(x)]$). It is quite evident that determiners in general fulfill this condition. E.g. from

(30) Most CEOs are incompetent.

follows

(31) Most CEOs are incompetent CEOs.

But are really all determiners conservative? *Only* is an apparent counterexample:

(32) Only CEOs are incompetent.

cannot be paraphrased as

(33) Only CEOs are incompetent CEOs.

(32) being contingent, (33) tautological. But besides this observation there are syntactical reasons to doubt the classification of *only* as a determiner. Other quantifying items whose conservativity is questioned are e.g. *many* and *few*.

The foundation for the generalization of quantifiers was in principle laid in Montague's work, but he himself did not refer to other quantifiers than the classical existential and universal quantifier. The theory of generalized quantifiers was recognized in linguistics in the earley 1980s, cf. Barwise & Cooper (1980) and article 4 [Semantics: Noun Phrases and Verb Phrases] (Keenan) *Quantifiers*.

10 Intensional theory of types

Montague developed his semantics as an intensional semantics, taking into account the non-extensional aspects of natural languages as alethic-modal, temporal and deontic operators. The intensional theory of types builds on Carnap's concept of intensions as functions from possible worlds to extensions. These functions are built into the type system by adding a new functional type from possible worlds s to the other types. Type s differs from the other types insofar as there are no expressions – constants or variables – directly denoting objects of this type, i.e. no specific possible worlds besides the contextually given current one can be addressed.

The difference between an extensional sentential operator like negation and an intensional like *necessarily* is reflected by their respective types: The meaning of the negation operator has type $\langle t, t\rangle$ while *necessarily* needs $\langle\langle s, t\rangle, t\rangle$ because not only the truth value of an argument p in the current world has impact on the truth value of *necessarily* p, but also the truth values in the alternative worlds.

In Montague (1973) he does not directly define a model-theoretic mapping for expressions of English, although this should be feasible in principle, but he gives a translation of English expressions into a logical language. Besides the usual ingredients of modal predicate logic, the lambda operator as well as variables and constants of the various types, he introduces the intensor ^ and extensor $^\vee$ of type $\langle T, \langle s, T\rangle\rangle$ and $\langle\langle s, T\rangle, T\rangle$ respectively for arbitrary types T. ^ transforms a given meaning into a Carnapian intension, i.e. $^\wedge a$ means the function from possible worlds to a's extensions in these worlds. In contrast, if b means an intension then $^\vee b$ refers to the extension in the current world. The intensor is used if a usually extensionally interpreted expression is used in an intensional context. E.g. *a unicorn* and *a centaur* mean generalized quantifiers, say Q_1 and Q_2, which extensionally are false for any argument. This may be different for other possible worlds. Therefore the intensions $^\wedge Q_1$ and $^\wedge Q_2$ may differ. This accounts for the fact that e.g. the intensional verb *seek* applied to $^\wedge Q_1$ and $^\wedge Q_2$ may result in different values, as

(34) John seeks a unicorn.

may be true while

(35) John seeks a centaur.

may be false at the same time. With the intensional extension of type logic, natural language semantics gets a powerful tool to account for the interaction of intensions and extensions in compositional semantics.

The same machinery which is applicable in alethic modal logic – i.e. the logic of possibility and necessity – is transferable to other branches of intensional semantics, as e.g. the semantics of tense and temporal expressions, cf. article 13 [Semantics: Noun Phrases and Verb Phrases] (Ogihara) *Tense*.

11 Dynamic logic

Natural language expresssions not only refer to time-dependent situations, but their interpretation also is dependent on time-dependent contexts. A preceding sentence may introduce referents for later anaphoric expressions (cf. article see article 12 [Semantics: Theories] (Dekker) *Dynamic semantics* and article 12 [Semantics: Interfaces] (Zimmermann) *Context dependency*).

(36) A man walks in the park. He whistles.

Among the anaphoric expressions are – of course – nominal anaphora, like pronouns and definite descriptions. Antecedents are typically indefinite noun phrases. But possible antecedents can also be introduced in a less obvious way, e.g. by propositions expressing events. The event time can then be referenced by a temporal anaphora like *at the same time*.

(37) The CEO lighted her cigarette. At the same time the health manager came in.

The anaphora *at the same time* refers to the time when the event described in the first proposition happens.

Anaphorical relations are not necessarily realized by overt expressions, they can be implicit, too, or they may be indicated by morphological means, e.g. by the choice of a grammatical tense. Anaphora poses a problem for compositional approaches to semantics based on predicate or type logic. (36) can be formalized by an expression headed by an existential quantifier like

(38) $\exists x[\text{man}(x) \land \text{w-i-t-p}(x) \land \text{whistle}(x)]$

But the man mentioned here can be referred to anywhere in the following discourse. Therefore the quantifier scope cannot be closed at any particular position in the discourse.

The semantics of anaphora, however, is not just a matter of the scope of existential quantifiers. Expressions, like indefinite noun phrases, usually meaning an existential quantifier can in certain contexts introduce discourse referents with a universal reading. This fact was described by Geach (1962).

(39) If a farmer owns a donkey he feeds it.

means

(40) $\forall x[\forall y[\text{farmer}(x) \land \text{donkey}(y) \land \text{own}(x, y) \rightarrow \text{feed}(x, y)]]$

This kind of anaphora is a further challenge for compositional semantics as it has to deal with the fact that an expression which is usually interpreted existentially gets a universal reading here. The challenge is addressed by dynamic semantics. Pre-compositional versions were developed independently by Hans Kamp and Irene Heim as Discourse Representation Theory and File Change Semantics respectively (cf. article 11 [Semantics: Theories] (Kamp & Reyle) *Discourse Representation Theory*). In both approaches structures representing the truth-conditional content of the parts of a discourse as well as the entities which are addressable anaphorically are manipulated by the meaning of discourse constituents. The answer to the question how the meaning of a discourse constituent is to be construed is simply: as a function from given discourse representing structures to new structures of the same type, cf. Muskens (1996).

This view is made explicit in dynamic logics. This kind of logics was developed in the 1970er by David Harel and others, cf. Harel (2000), and has been primarily used for the formal interpretation of procedural programming languages.

(41) $\langle a \rangle q$

means that statement a possibly leads to a state where q is true, while

(42) $[a]q$

means that statement a necessarily leads to a state where q is true. Regarding states as possible worlds we arrive at a modal logic with as many modalities as there are (equivalence classes of) statements a, for a recursive language usually infinitely many. For many purposes the consideration can be constrained to such modalities where from each state exactly one successor state is accessible. For such modalities with functional accessibility relation the weak (⟨...⟩) and the strong operator ([...]) collapse semantically into one operator. If we further agree that

(43) $s_1 \wedge [a]s_2$

can be rewritten as

(44) $s_1[a]s_2$

then this notation has the intuitive reading that state s_1 is mapped by the meaning of a into s_2. A simplistic application is the following: Assume that s_1 is a characterization of the knowledge state of a recipient before receiving the information given by assertion a. Then s_2 is a characterization of the knowledge state of the recipient after being informed. If you identify knowledge states with those sets of possible worlds which are consistent with the current knowledge, and if you consider s_1 and s_2 as descriptions of sets W_1 and W_2 of possible worlds, then they differ in exactly that respect that W_2 is the intersection of W_1 and the set W_a of possible worlds in which a is true, i.e. $W_2 = W_1 \cap W_a$.

The notion of an informative utterance a can be defined by the condition that $W_2 \neq W_1$. And in order to be consistent with the previous context, it must be true that $W_2 \neq \varnothing$. The treatment of discourse states or contexts in dynamic logics is not limited to truth-conditionally characterizable knowledge. In principle any kind of linguistic context parameters can be part of the states, among these the anaphorically accessible antecedents of a sentence.

In their Dynamic Predicate Logic, as developed in Groenendijk & Stokhof (1991), Groenendijk and Stokhof model the anaphoric phenomena accounted for in Kamp's Discourse Representation Theory and Heim's File Change Semantics in a fully compositional fashion. This is mainly achieved by a dynamic interpretation of the existential quantifier:

(45) $\exists x[P(x)]$

is semantically characterized by the usual truth conditions but has the additional effect that free occurrences of the variable x have to be kept assigned to the same object in subsequent expressions which are connected appropriately. The dynamic effect is limited by the scopes of certain operators like the universal quantifier, negation, disjunction, and implication. So x is bound to the same object ouside the syntactic scope of the existential quantifier as in

(46) $\exists x[\text{man}(x) \wedge \text{w-i-t-p}(x)] \wedge \text{whistle}(x)$

although the syntactic scope of the existential quantifier ends after w-i-t-p(x). This propsosition is true, only if there is an object x which verifies all three predicates man, w-i-t-p, and whistle.

If we characterize the dynamic dimension, the sentences (36) and (39) can be formalized in Dynamic Predicate Logic as

(47) $\exists x[\text{man}(x) \wedge \text{w-i-t-p}(x)] \wedge \text{whistle}(x)$

and

(48) $\exists x[\text{farmer}(x) \wedge \exists y[\text{donkey}(y) \wedge \text{own}(x, y)]] \rightarrow \text{feed}(x, y)$

respectively. It can easily be seen, how the usual meanings of the discourse sentences

(49) A man walks in the park.

and

(50) A farmer has a donkey.

enter into the composed meaning of the discourse without any changes. In order to get the intended truth conditions for implications, it is required as truth condition that the second clause can be verified for any assigment to *x* verifying the first clause.

Putting together the filtering effect of propositions on possible worlds and the modifying effect on assignment functions, we can consider propositions in Dynamic Predicate Logic as functions on sets of world-assigment pairs. The empty context can be characterized by the Cartesian product of the set of possible worlds and the set of assigments. Each proposition of a discourse filters out certain world-assignment pairs. In some respects Dynamic Predicate Logic deviates from standard dynamic logic approaches, as Groenendijk & Stokhof (1991, sec. 4.3) point out, but it still can be seen as a special case of a logic in this framework.

The dynamic view of semantics can be used to model other contextual dependencies than just anaphora. Groenendijk (1999) shows another application in the Logic of Interrogation. Questions add felicity conditions for a subsequent answer to the discourse context. To a great extent, these conditions can be characterized semantically. In the Logic of Interrogation the effect of a question is understood as a partitioning of the current set of possible worlds. Each partition stands for some alternative exhaustive answer, e.g. a yes-no question partitions the set of possible worlds into one subset consistent with the positive answer and a complementary subset compatible with the negative answer.

Let us e.g. take the question (51).

(51) Does a man walk in the park?

According to the formalism of Groenendijk (1999) (51) can be formalized as (52).

(52) $?\exists x\,[\text{man}(x) \wedge \text{w-i-t-p}(x)]$

(52) partitions the set of possible worlds in two subsets W^+ and W^-, such that (53) is true for every world in W^+ and false for every world in its complement subset W^-. An appropriate answer selects exactly one of these subsets.

(53) $\exists x[\text{man}(x) \wedge \text{w-i-t-p}(x)]$

Wh-questions like (54) partition the sets of possible worlds in more partitions than yes-no questions.

(54) Who walks in the park?

Each partition corresponds to an exhaustive answer, which provides the full information who walks in the park and who does not. An assertion answers the question partially if it eliminates at least one partition. If it furthermore eliminates all partitions but one, it answers the question exhaustively.

This chapter has given a short overview how logic provided the tools for treating linguistic phenomena. Each logical system has its characteristic strength and its limits. The limits of a logical system sometimes inspired the development of new formal tools (e.g. the step from Aristotelian syllogistic to modern predicate logic) which inspired a new semantics. Sometimes a change in the focus of linguistic phenomena inspired a systematic search for new logical tools (e.g. modal logic) or a reinterpretation of already available logical tools (e.g. dynamic logic). We hope to have illustrated the main developments of the bi-directional influences of logical systems and semantics.

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